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Competing risks mixture model for traffic incident duration prediction

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ABSTRACT

Traffic incident duration is known to result from a combination of multiple factors, including covariates such as spatial and temporal characteristics, traffic conditions, and existence of secondary accidents but also the clearance method itself. In this paper, a competing risks mixture model is used to investigate the influence of clearance methods and various covariates on the duration of traffic incidents and predict traffic incident duration. The proposed mixture model considers the uncertainty in any of five clearance methods that occurred. The probability of the clearance method is specified in the mixture by using a multinomial logistic model. Three candidate distributions, namely, generalized gamma, Weibull, and log-logistic are tested to determine the most appropriate probability density function of the parametric survival analysis model. The unobserved heterogeneity is also incorporated into the mixture model in a way that allows parameters to vary across observations based on the three candidate distributions. The methods are illustrated with incident data from Singaporean expressways from January 2010 to December 2011. Regression analysis reveals that the probability of different clearance methods and the duration of traffic incidents are both significantly affected by various factors, such as traffic conditions and incident characteristics. Results show that the proposed mixture model is better than the traditional accelerated failure time model, and it predicts traffic incident duration with reasonable accuracy, as shown by the mean average percent error.

1. Introduction

Traffic incidents are major causes of non-recurrent congestion on expressways (Haas, 2006) and urban arterial roads (Shao and He, 2008). An effective approach to reduce the influence of traffic incidents is the application of the Traffic Incident Management System (TIMS), which requires a timely and precise estimation of traffic incident duration. By performing reliable prediction of incident duration, traffic operators could deploy appropriate measures around the incident location and provide travelers with real-time traffic information to reduce incident-related traffic congestion. In the past two decades, significant research effort has been exerted on the analysis and prediction of traffic incident duration.

Total incident duration can be divided into the following sequential and distinct time intervals (Nam and Mannering, 2000; TRB, 2000; Valenti et al., 2010):

- Detection/reporting time: time between the time of incident occurrence and the time of response by the traffic control center operators receiving the call; usually, this period is difficult to capture.
- Preparation/dispatching time: time between operators receiving the call and dispatching the incident response team members.
- Travel time: time between incident response team members receiving the dispatch order and their arrival at the incident location.
- Clearance time: time between the incident response team members' arrival and incident clearance.

A general assumption in this work is that a model of incident duration can be discretized according to groups of "clearance methods", namely related to participation of police, tow truck or

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drivers self-driving off of the scene. From the perspective of hazard-based modeling, these methods correspond to "failure events". Since the clearance method is not always observable, we propose a discrete mixture model.

This paper uses a hazard-based competing risks mixture model and is focused on analyzing the influence of various factors on the incident duration, as determined from the moment the operators receive the call to the total incident clearance, when the roadway capacity returns to its normal conditions; this duration is the sum of total preparation time, travel time, and clearance time but excludes the unknown period from the time of incidence occurrence to the time of reporting. The proposed model is also tested to predict the traffic incident duration.

The rest of this paper is organized as follows. The first section presents a literature review of online incident duration predictions and hazard-based models. The next section discusses the proposed parametric mixture approach used to analyze the duration of traffic incidents; parametric mixture duration models and coefficient estimation methods are also discussed in this section. The third section describes the data used in this study. The fourth section presents the calibration of the parametric mixture model, and the fifth section evaluates the prediction accuracy. Lastly, we end this paper by presenting our conclusions and suggesting future plans.

2. Literature review

In the past two decades, traffic incident duration has been investigated through various approaches, such as finding the factors that significantly affect traffic incident duration (Khattak et al., 2010; Zhang and Khattak, 2010) or predicting traffic incident duration. The majority of the literature review is focused on the latter.

Various regression models have been applied in predicting traffic incident duration. Khattak et al. (1995) applied a truncated regression model based on a simple time sequential procedure to predict the traffic incident duration by establishing a relationship between traffic incident duration and independent variables. However, they did not examine the prediction accuracy of the sequential model because not enough data were available to support the test. Peeta et al. (2000) developed a linear regression model that predicts traffic incident clearance time with time-independent variables. He et al. (2011) established an incident duration prediction model based on hybrid tree-based quantile regression to predict traffic incident duration on urban freeways. The results of the study showed that the proposed model had better prediction performance in comparison with three other kinds of prediction models. A non-parametric regression model (Smith and Smith, 2001) has also been used to predict traffic incident duration; however, the performance of the model was unsatisfactory, with an average error of more than 20 min.

Decision trees and classification trees, such as the classification and regression tree (Kim et al., 2008; Knibbe et al., 2006) and the M5P tree (Zhan et al., 2011), have also been applied to predict traffic incident duration. One study was able to predict incident duration on the basis of the time interval it occurred and obtained an overall confidence of over 80% (Kim et al., 2008). Another study showed that the M5P tree algorithm can perform a prediction with a mean average percentage error (MAPE) of 42.7% (Zhan et al., 2011).

Several kinds of Bayesian classifiers (Boyles et al., 2007; Demiroluk and Ozbay, 2011; JiYang et al., 2008; Li and Cheng, 2011; Shen and Huang, 2011) have been used to accommodate incomplete information or information received at different time points. These studies show that the Bayesian classifier has better prediction performance than other traditional models (Demiroluk and Ozbay, 2011), such as linear regression and classification and regression trees (CART); for example, the presented model outperformed the CART model with 53% accuracy rate.

A number of studies recently applied artificial neural networks (ANN) in developing prediction models of traffic incident duration. Wei et al. (Lee and Wei 2010; Wei and Lee, 2007) developed two ANN-based models that sequentially predict traffic accident duration, and the results showed that these models achieve a reasonable prediction; that is, the MAPEs of the models were mostly under 40% (Wei and Lee, 2007). Pereira et al. (2013) used radial basis function algorithm, which continuously makes predictions as new information arrives. New information arrives in the form of text messages (internal to the traffic operator and emergency response system) and is analyzed with text mining techniques (latent Dirichlet allocation) to extract a list of "topics" associated with the current situation of the incident. The overall median error decreased by 28% in the approach with topics in comparison with that without topics. Other techniques, such as genetic algorithms (GA) (Lee and Wei 2010) and fuzzy logic (Vlahogianni and Karlaftis, 2013) have been combined with ANN to obtain better prediction performance. Vlahogianni and Karlaftis (2013) applied fuzzy entropy feature selection to select the factors to be used for incident duration prediction. Hazard-based duration models, which focus on time to event data, have been used in estimating and predicting traffic incident duration. Jones et al. (1991) applied an accelerated failure time (AFT) model with log-normal distribution to examine the factors affecting the incident duration on Seattle freeways. AFT hazard-based duration models were then applied to different traffic incident duration intervals (Nam and Mannering, 2000), and the results revealed that different distributions of the hazard function are suitable for different incident duration intervals and that a wide variety of factors significantly affect incident time intervals.

Different distributions-based AFT hazard-based models, which are based on different data resources of traffic incidents, have been used to estimate and predict traffic incident duration. These models include log-logistic distribution (Chung, 2010; Chung et al., 2010; Kang and Fang, 2011; Wang et al., 2013), Weibull distribution (Alkaabi et al., 2011; Hojati et al., 2013), log-normal distribution (Chung and Yoon, 2012) and gamma distribution (Li, 2014). To find a more appropriate distribution for the hazard function, Ghosh et al. (2012) applied the flexible generalized F distribution to fit the traffic incident duration.

On basis of whether the model distinguishes multiple (and possibly latent) clearance methods, two types of hazard-based models can be considered: single and competing risks hazard-based models. The above mentioned models generally fall under the single risk category. However, in view of the high heterogeneity in incident types, driver behavior (i.e., the drivers that participate in the incident), and response strategies, significant information may be lost when all of the factors are aggregated into one type.

Competing risks hazard-based models have recently been widely used in medical research (Fürstová and Valenta, 2011; Haller et al., 2013; Lau et al., 2008, 2009, 2011; Ravani et al., 2005) and in transportation, among other fields. Gilbert (1992),Hensher (1998), and Yamamoto et al. (2004) applied competing risks hazard-based models to investigate the time spent and influencing factors in automobile transactions. Ettema et al. (1995) and Bhat (1996) investigated travel activity duration by using accelerated lifetime and proportional hazard models, respectively. Li and Guo (2014) investigated the factors effected on the duration of two incident group with proportional hazard competing risk model. Shyr and Ben-Akiva (1996) used mixture competing risks hazard models to examine rail fatigue behavior. The use of competing risks mixture models in analyzing and predicting incident data has not been indicated in the literature.

3. Competing risks model

3.1. Mixture model

Competing risks occur when an incident can be cleared by at least two possible ways but only one can actually occur. In classical competing risks, the observed outcome dataset is represented by (T, C, X), where T is the time to failure, assumed to be a continuous and positive random variable, C is the cause of failure and takes one of the values in the finite set $\{1,2, \ldots \ldots\}$, and X is a vector of N covariates. For an individual $i = 1,2, \ldots \ldots M$, the data record is (T, C, X).

This study applied a mixture model of competing risks (Larson and Dinse, 1985), where several event types and times are modeled through a joint distribution, broken down into a sum (mixture) of individual distributions, each one corresponding to a potential event type. In our case, these failures are mutually exclusive and discrete, so we represent it as a multinomial logit (MNL) function.

In fact, for different incidents, the response patterns will also be different. For some accidents, such as vehicle overturns or accidents involving injures, there should be an incident response team (for example traffic police) to evaluate who is held accountable for the accident, save the wounded and/or tow the stalled vehicles. In comparison, for some minor accidents, such as those without injuries or vehicles that are still functional, the traffic management department encourages the drivers involved to negotiate among themselves before the incident response team arrives at the scene, as well as fill in the necessary insurance forms and take photos for evidence to reduce the incident duration, which is similar with the quick clearance policy enforced in some countries (Owens et al., 2010).

In this study, the events of failure are five types of incident clearance methods: clearance method 1, drive self without police arrival (drivers involved in an incident will drive off without police arrival); clearance method 2, drive self with police arrival (drivers involved in an incident will drive off after the police arrives); clearance method 3, tow without police arrival (some or all of the vehicles involved in an incident will be towed without police arrival); and clearance method 4, tow with police arrival (some or all the vehicles involved in an incident will be towed and with police arrival). Other incidents are cleared by unknown or uncertain clearance methods and are considered classified under clearance method 5.

The cumulative incidence function (CIF) for the kth type of failure is:

$$\begin{aligned} F_k^*(t) &= \Pr(T \le t, C = k) = \Pr(C = k)\Pr(T \le t | C = k) \\ &= \Pr(C = k) F_k(t) \end{aligned} \tag{1}$$

The overall distribution function is the sum of CIFs. Thus,

$$F(t) = \Pr(T \le t) = \sum_{k=1}^{K} F_k^*(t) \tag{2}$$

Similarly, the overall survivor function is the sum of k sub-survivor functions, where each sub-survivor is the survivor function for the kth type of failure. Therefore,

$$S(t) = \Pr(T > t) = \sum_{k=1}^{K} S_k^*(t) \tag{3}$$

where $S_k^*(t)$ is the sub-survivor function for kth type of failure.

The corresponding sub-density function is

$$f_k^*(t) = \frac{\partial F_k^*(t)}{\partial t} = \frac{\mathrm{d}\Pr(T \le t | C = k)}{\mathrm{d}t} \Pr(C = k) = f_k(t)\Pr(C = k) \tag{4}$$

and the mixture model is

$$f(t) = \sum_{k=1}^{K} f_k^*(t) = \sum_{k=1}^{K} \Pr(C = k) f_k(t) \tag{5}$$

With covariate vector X, Eqs. (2) and (5) and can be rewritten as follows:

$$F(t|X) = \sum_{k=1}^{K} F_k^*(t|X) = \sum_{k=1}^{K} F_k(t|X)\Pr(C = k|X) \tag{6}$$

$$f(t|X) = \sum_{k=1}^{K} f_k^*(t|X) = \sum_{k=1}^{K} f_k(t|X)\Pr(C = k|X) \tag{7}$$

A multinomial logistic (MNL) regression model is used to assess the influence of covariates on the probability of failing from a certain cause k (Haller et al., 2013); this model is given by

$$\pi_k = \Pr(C = k|X) = \frac{\exp(\mu_k + \phi_k^T X)}{\Sigma_{l=1}^{K} \exp(\mu_l + \phi_l^T X)} \tag{8}$$

where μ_k is a scalar constant, and ϕ_k^T is a row vector of N regression coefficients $(k = 1,2, \ldots \ldots K)$. For $(k = 1,2, \ldots \ldots K-1)$, Eq. (8) can be written as

$$\pi_k = \Pr(C = k|X) = \frac{\exp(\mu_k + \phi_k^T X)}{1 + \Sigma_{l=1}^{K-1} \exp(\mu_l + \phi_l^T X)} \tag{9}$$

and

$$\pi_k = 1 - \sum_{i=1}^{k-1} \pi_i \tag{10}$$

3.2. Maximum likelihood estimation

In a mixture model, the contribution to the likelihood function of an incident i with covariate vector X_i with a type k failure at time t_i is (Lau et al., 2008):

$$L_i = \pi_{ki} f_k(t_i) \tag{11}$$

where

$$\pi_{ki} = \Pr(C_i = k|X_i)$$

All of the traffic incidents in the research dataset have been cleared, and no right-censored data are present. Thus, for the known four clearance methods, the likelihood function is

$$\begin{aligned} L_i = &[\pi_{1i} \cdot f_1(t_i)]^{\delta_{1i}} \times [\pi_{2i} \cdot f_2(t_i)]^{\delta_{2i}} \times [\pi_{3i} \cdot f_3(t_i)]^{\delta_{3i}} \\ &\times [(1 - \pi_{1i} - \pi_{2i} - \pi_{3i}) \cdot f_4(t_i)]^{\delta_{4i}} \end{aligned} \tag{12}$$

and the indicator function is given by

$$\delta_{ki} = \left\{ \begin{matrix} 0 & \text{if} & C_i \ne k \\ 1 & \text{if} & C_i = k \end{matrix} \right\} \tag{13}$$

where $\pi_{ki} = \Pr(C_i = k|X_i)$ is the mixture probability and $f_k(t_i)$ is the probability density function for the parametric distribution for a type k failure.

In Eq. (12), the clearance method for each incident is assumed to be known. However, in reality, the time of clearance may be known while the clearance method used is uncertain. The actual clearance method cannot be measured with certainty or be completely ascertained. Instead of eliminating or censoring those data, the above likelihood may be modified to incorporate the uncertainty of

Table 1
Statistical characteristics of the selected incident database.

Method	Obs number	Min. (min)	Max. (min)	Mean (min)	Std. dev	Variance	Skewness	Kurtosis
1- Self drive without police	1474	2	147	11.55	11.30	127.84	3.90	30.20
2- Self drive with police	237	2	154	29.43	25.22	636.41	1.81	7.24
3- Tow without police	2224	2	170	32.63	27.28	744.27	1.90	7.15
4- Tow with police	3629	2	180	55.82	35.79	1281.14	1.05	3.72
5- Unknown method	4529	2	176	26.78	28.70	823.63	1.82	6.67

clearance method type by obtaining the sum of $f_k(t)$ for all events that could have occurred, such that the likelihood in Eq. (12) would be

$$\begin{aligned} L_i = {} & [\pi_{1i} \cdot f_1(t_i)]^{\delta_{1i}} \times [\pi_{2i} \cdot f_2(t_i)]^{\delta_{2i}} \times [\pi_{3i} \cdot f_3(t_i)]^{\delta_{3i}} \\ & \times [(1-\pi_{1i}-\pi_{2i}-\pi_{3i}) \cdot f_4(t_i)]^{\delta_{4i}} \\ & \times [\pi_{1i} \cdot f_1(t_i) + \pi_{2i} \cdot f_2(t_i) + \pi_{3i} \cdot f_3(t_i) \\ & + (1-\pi_{1i}-\pi_{2i}-\pi_{3i}) \cdot f_4(t_i)]^{1-\delta_{1i}-\delta_{2i}-\delta_{3i}-\delta_{4i}} \end{aligned} \tag{14}$$

3.3. Hazard function distribution

According to previous studies, Weibull distribution (Alkaabi et al., 2011; Hojati et al., 2013; Nam and Mannering, 2000) or log-logistic distribution (Chung, 2010; Chung et al., 2010; Hu et al., 2011; Jones et al., 1991; Qi and Teng, 2008; Wang et al., 2013) is the best distribution for traffic incident duration. Thus, these distributions were tested in this study. To find other possible distributions for the incident duration, the generalized gamma distribution was also used to estimate $f_k(t_i)$ and their respective survivor functions.

In this study, the conditional clearance time distributions for all methods were modeled with a three-parameter generalized gamma distribution GG(β, σ, λ); these parameters are location (β), scale (σ), and shape (λ). The generalized gamma distribution can be reduced to commonly used distributions, such as exponential when $\lambda = \sigma = 1$, gamma when $\lambda = \sigma$, log-normal when $\lambda = 0$, and Weibull distribution when $\lambda = 1$ (Cox et al., 2007; Lau et al., 2008). The mixture of generalized gamma distributions makes the model flexible.

The probability density function $f_k(t_i)$ and the survival function $S_k(t_i)$ are defined as follows (Lau et al., 2008):

$$f_k(t_i) = \frac{|\lambda_k|}{\sigma_k t_i \Gamma(\lambda_k^{-2})} [\lambda_k^{-2} (e^{-\beta_k} t_i)^{\lambda_k/\sigma_k}]^{\lambda_k^{-2}} \exp[-\lambda_k^{-2} (e^{-\beta_k} t_i)^{\lambda_k/\sigma_k}] \tag{15}$$

$$S_k(t_i) = \begin{Bmatrix} 1 - \Gamma[\lambda_k^{-2}(e^{-\beta_k} t_i)^{\lambda_k/\sigma_k}; \lambda_k^{-2}] & \text{if} \quad \lambda_k > 0 \\ \Gamma[\lambda_k^{-2}(e^{-\beta_k} t_i)^{\lambda_k/\sigma_k}; \lambda_k^{-2}] & \text{if} \quad \lambda_k < 0 \end{Bmatrix} \tag{16}$$

where $\Gamma[\ldots;\ldots]$ is the cumulative distribution function for the two-parameter gamma distribution, with mean and variance equal to $\gamma > 0$; that is, $\Gamma(t, \gamma) = \int_0^t y^{\gamma-1} e^{-y} dy / \Gamma(\gamma)$ (Cox et al., 2007; Lau et al., 2008).

This study assumed that the scale and shape parameters do not change, whereas the location parameter is a linear combination of the covariates; thus, the generalized gamma distributions are similarly parameterized to a conventional regression model with $GG(\alpha + \beta_k^T X, \sigma, \lambda)$, in which β_k^T is a vector of estimable parameters.

3.4. Unobserved heterogeneity

In the above fixed parameter models, it is assumed that all differences between traffic incidents were captured with the observed explanatory variables (the vector), that is, the effect of any individual explanatory variable is the same for each observation. But in fact, it is difficult to obtain all the related information that truly influences the duration of traffic incidents. For example, there may be unobservable factors influencing the traffic incident duration, such as traffic flow volume, that cannot be integrated into the incident dataset. Consequently it is necessary to consider unobserved factors that are not included in the covariates vector, which is usually referred to as unobserved heterogeneity. Two approaches can be used to examine the heterogeneity assumption, namely, applying the gamma distribution to incorporate heterogeneity and allowing parameters to vary across observations based on some pre-specified distribution, known as the random-parameter duration model (Anastasopoulos et al., 2012; Anastasopoulos and Mannering, 2014; Washington et al., 2011). For the context of traffic incidents, the random-parameter duration model has been reported to outperform the fixed parameter duration models (Hojati et al., 2013, 2014). In this paper, we will follow such suggestion and introduce random parameters in our duration models (Anastasopoulos and Mannering, 2014; Hojati et al., 2014)[1], from now on referred to as random parameter models. We assume that, for each observation i (Washington et al., 2011), we have

$$\beta_i = \beta + \omega_i \tag{17}$$

where β_i is a vector of parameters that vary across M incidents and ω_i is a randomly distributed term (e.g., normally distributed term with mean zero and variance σ^2). In this paper we follow earlier research (Hojati et al., 2014) in using the normal distribution directly and did not compare it with some other distributions.

4. Data description

The data used in this study consisted of 12,093 incident records with duration of more than two minutes on Singapore expressways from January 2010 to December 2011. Each incident record originated from external information (driver's call, police, etc.) and created in the traffic management center in Singapore. The incident response adapts to the situation when new information continually becomes available. The initial information includes location (road name, coordinates, distance to on/off ramp, zone ID), direction, traffic condition (congestion/non congestion), number of affected lanes and vehicles, etc. In addition to time tag, additional new information are inserted in an incident record in text form.

The characteristics of the incident database are shown in Table 1. Overall, the database can be divided into five groups. The duration of incident with clearance method 4 is longest[2]. The skewness values are greater than zero, indicating that the traffic incident duration in the groups are all right-skewed. The distributions of the groups are shown in Fig. 1. The distribution differs for each incident record group. Curiously, methods 2

[1] It is worth noting that other options exist, such as modeling together random parameters model with gamma heterogeneity, as in (Anastasopoulos and Mannering, 2014).

[2] We should note the risk of endogeneity between incident duration and likelihood of police arrival, as there is a potential circular causality between police clearance process and extended duration. Hence, the results presented should be considered with this aspect in mind.

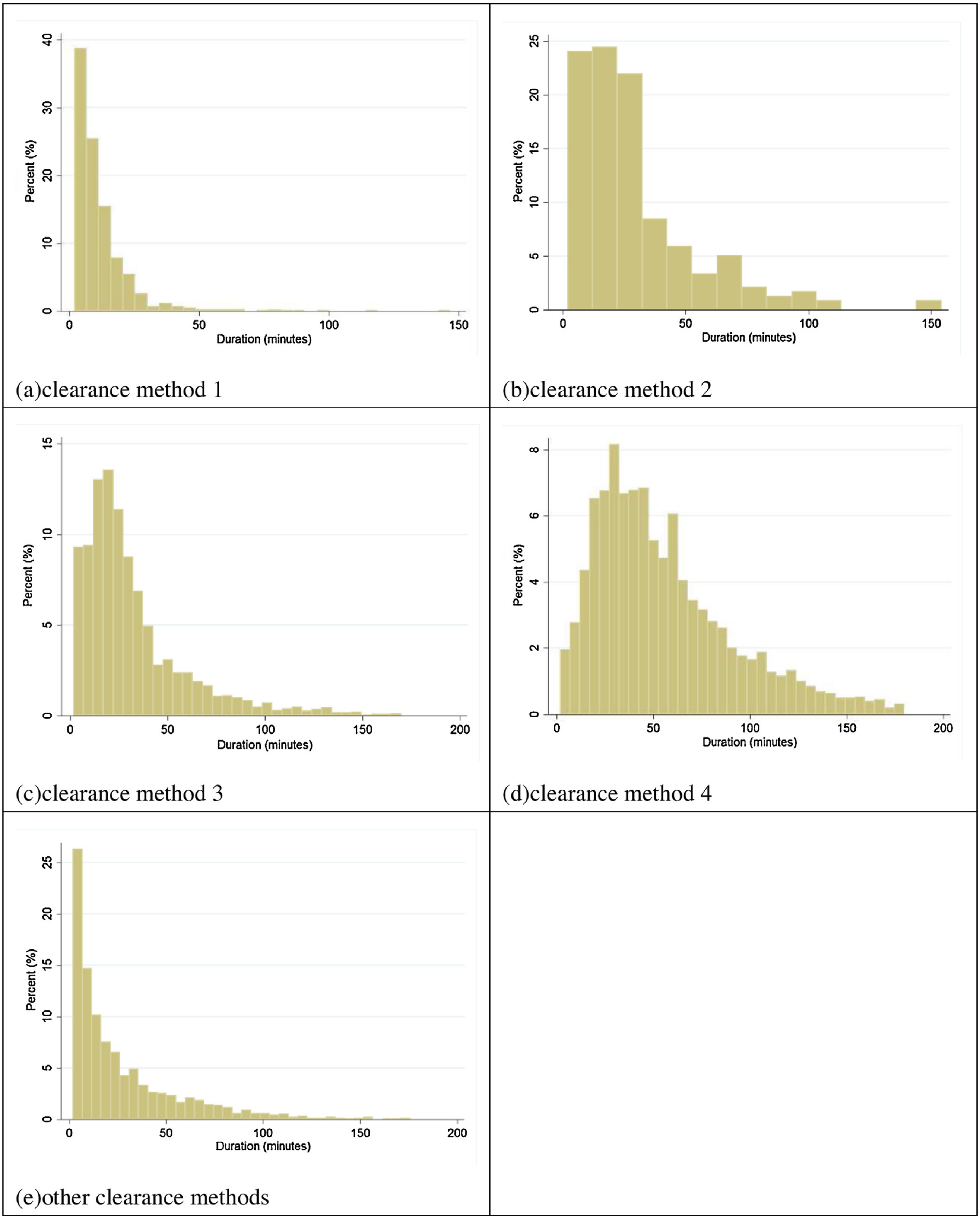

Fig. 1. Distribution of different incident record groups.

(self drive with police) and 3 (tow without police) are the most similar to each other despite their apparent un-relatedness. Except for these two cases, the fact that these distributions are generally different support the idea of a competing risks mixture model, that assumes that duration distribution is generated through different underlying processes.

Kaplan–Meier estimation is a non-parametric method of estimating $S(t)$ from data. The Kaplan–Meier (KM) estimates

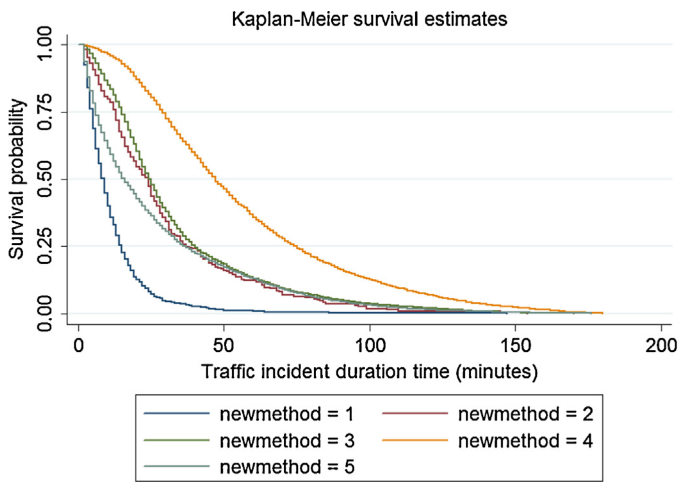

Fig. 2. The Kaplan–Meier plots of five clearance methods.

of the survival function (Kaplan and Meier, 1958) of the five clearance methods are shown in Fig. 2.

By using Fig. 2, we can compare the curves for the five different groups of incidents with clearance methods. The KM curve for incidents with clearance method 4 is consistently higher than the KM curve for clearance method 1, suggesting that the incidents that needed towing and police arrival have longer duration than the incidents where those involved drove off before the police arrived.

For all five KM curves, gaps in these curves can be found in an horizontal or vertical direction. A vertical gap means that, at a specific point in time, one group from the upper curve had a greater fraction of incidents that had not yet been cleared. A horizontal gap means that it took longer for one group from the right curve to experience a certain fraction of clearance.

Research over the past few decades has elucidated the relationships between influencing variables and incident duration. Several reported variable groups affecting incident duration include incident characteristics (e.g., incident type and severity; number and type of vehicles involved; number of injures or fatality) (Chung, 2010; Zhan et al., 2011), temporal characteristics (Chung, 2010), traffic condition (Kim and Chang, 2012; Vlahogianni and Karlaftis, 2013), road geometric characteristics (Jones et al., 1991), operational factors (Khattak et al., 1995; Kim and Chang, 2012), environmental characteristics (Nam and Mannering, 2000; Vlahogianni and Karlaftis, 2013), and textual topics from incident reports (Pereira et al., 2013). In this study, the candidate variables shown in Table 2 were extracted from the initial information and the reported textual information of an incident.

The candidate variables derived from two types of data are the set of values created by the operator (location, lanes blocked, etc.) and the incident characteristics extracted from the incident report. Following the recommendation of Khattak et al. (2012) on the interactions between different incidents that are close in time and space, we computed a few factors that should contribute to incident duration; that is, for each record, we calculated the number of incidents that occurred at several distances (same road, 100 m, 1000 m, 5000 m) during a time window before the current incident.

From the continual textual information before an incident clearance, we also identified various incident characteristics, such as whether the incident involved bikes, fire, injuries, buses, trucks, motorcycles, etc. A simplistic capacity reduction value was calculated by dividing the number of affected lanes by the total number of lanes in the affected area; however, this approach requires a more detailed analysis in future studies.

5. Model development

A total of 8062 incident records were chosen to estimate the proposed competing risk mixture model, and 4031 records were used to test the prediction accuracy of the developed model. The former, training set, corresponds to earlier dates than the latter.

Analysis was performed with the statistical software SAS where Eq. (14) is the log-likelihood (SAS Institute Inc., 2013). For a parametric model application, the most appropriate distribution for the duration must be identified by assessing the goodness of fit in terms of various measures such as likelihood (Ghosh et al., 2012; Nam and Mannering, 2000), Akaike's information criterion (AIC) (Hojati et al., 2013; Wang et al., 2013), or Bayesian information criterion (BIC). Table 3 shows the results of the different distributions.

Table 3 shows that the random parameter log-logistic distribution is the best fit for the incident duration although with

Table 2
Potential variables and their descriptions.

Variable	Value
Temporal characteristics	
Weekday	Binary variable: 1—weekday; 0—weekend
Spatial characteristics	
Same road	Continuous variable: the incident number on the same road during a time window
Less 100	Continuous variable: the incident number in 100 m during a time window
Less 1000	Continuous variable: the incident number in 1000 m during a time window
Less 5000	Continuous variable: the incident number in 5000 m during a time window
Traffic condition	
Congestion	Binary variable: 1—congested traffic condition when incident occurrence; 0—no-congested traffic condition
Capacity reduction	Continuous variable: percent
Block shoulder	Binary variable: 1—the shoulder is blocked; 0—the shoulder is blocked
Incident characteristics	
Bike	Binary variable: 1—incident involve bike; 0—no bike
Ambulance	Binary variable: 1—incident need ambulance; 0—no need ambulance
Fire	Binary variable: 1—incident involve fire; 0—no fire
Injure	Binary variable: 1—incident involve injure; 0—no injure
Taxi	Binary variable: 1—incident involve taxi; 0—no taxi
Bus	Binary variable: 1-–incident involve bus; 0-–no bus
truck	Binary variable: 1—incident involve truck; 0—no truck
Motorcycle	Binary variable: 1—incident involve motorcycle; 0—no motorcycle

Table 3
Goodness of fit of different models.

Distribution	Model type	−2 log likelihood	AIC	BIC
Generalized gamma	Fixed parameters	85356	85610	86500
	Random parameters	78084	78340	78173
Weibull	Fixed parameters	79990	80236	81098
	Random parameters	77811	78062	77896
Log-logistic	Fixed parameters	79343	79589	80450
	Random parameters	77518	77766	77604

a marginal advantage over Weibull, similarly to previous research (Chung, 2010; Chung et al., 2010; Hu et al., 2011). For the remainder of this paper, a competing risks mixture model based on random parameter log-logistic distribution is estimated and used to predict the duration of incidents.

It could be argued that, by splitting dataset for estimation (only 2/3 are used for training), our results will be less robust, so we re-estimated the model with the full dataset and the results show that the differences in the estimated coefficients are marginal. This indicates that our dataset is large enough for model estimation and validation with unseen data, following a similar approach as some previous studies (Chung, 2010; Wei and Lee, 2007; Zhan et al., 2011).

The estimated coefficients of the proposed competing risks mixture approach based on log-logistic distribution with random parameters are presented in Table 4. The multinomial logistic model and the hazard-based models' regression coefficients were simultaneously estimated. Therefore, useful interpretations were obtained as to how various factors affect the incidence of each clearance method and how they influence the clearance time by each method. On the other hand, some random parameters present standard deviation values that are significantly different from zero, that is, the effect of some variables does vary across the observations. Notice that for significant variables only random parameters that have standard deviations that are significantly different from zero have been shown in the table.

The analysis of the coefficients for the clearance methods logistic regression needs to consider also method 4 (both police and tow vehicle present), which is here the "default" method. For example, a positive coefficient μ_k of the logistic regression, which models the expected type of clearance, indicates that this incident type is more likely to be cleared by clearance method k than method 4. In the following section we only analyze the factors that significantly affect the clearance methods and duration time.

The coefficients in column 2 of Table 4 indicate that the incident on weekdays were more likely to be cleared by method 1 (drive self without police). With the increase in the number of incidents that occur on the same road within a time interval, the current incident is less likely to be cleared by method 1 than method 4. Incidents under congested conditions, or with capacity reduction, shoulder blocking, bikes, ambulances, fire, injuries, taxis, trucks, or motorcycles, are less likely to be cleared by clearance method 1 than method 4; that is, these kinds of incidents are usually more severe in comparison with the absence of these factors and are more likely to need towing and police arrival.

For clearance method 2, the coefficients of the logistic regression in column 3 of Table 4 indicate that incidents with bikes, ambulances, injuries, or motorcycles are less likely to be cleared by method 2 than method 4.

The coefficient of variable weekday means that incidents on a weekday are more likely to be cleared by method 3 (tow without police) than clearance method 4, because the drivers may have no time to wait for the police to arrive (Table 1 shows that the mean duration of method 3 is approximately half that of method 4). Under congestion conditions, incidents are more likely to be cleared by method 3. Similar to clearance method 1, incidents with capacity reduction, shoulder blocking, bikes, ambulances, fire,

Table 4
Maximum likelihood estimates (with t-stat in parenthesis) for competing risk mixture method (* denotes P-value < 0.05).

	Clearance methods			Duration			
	Method 1	Method 2	Method 3	Method 1	Method 2	Method 3	Method 4
	$\hat{\pi}_1 T$and $\hat{\mu}_1$	$\hat{\pi}_2 T$and $\hat{\mu}_2$	$\hat{\pi}_3 T$and $\hat{\mu}_3$	$\hat{\alpha}_1, \hat{\lambda}_1 T, \hat{\gamma}_1$	$\hat{\alpha}_2, \hat{\lambda}_2 T, \hat{\gamma}_2$	$\hat{\alpha}_3, \hat{\lambda}_3 T, \hat{\gamma}_3$	$\hat{\alpha}_4, \hat{\lambda}_4 T, \hat{\gamma}_4$
Contant ($\hat{\mu}_1$ or $\hat{\alpha}$)	2.4495 (15.02)*	−1.2621 (-3.84)*	1.1052 (8)*	0.7016 (0.95)	1.3655 (1.76)	1.9506 (2.65)*	2.8898 (3.94)*
Weekday	0.5321 (5.55)*	0.2532 (1.3)	0.293 (3.38)*	−0.0157 (−0.35)	0.036 (0.21)	−0.0299 (−0.69)	−0.0327 (−1.17)
Sameroad	−0.1608 (−2.34)*	0.0746 (0.69)	−0.041 (−0.75)	0.0592 (1.55)	0.0007 (0.01)	0.0485 (1.97)*	0.0162 (0.99)
Standard deviation[a]				– –	– –	0.032 (3.26)	– –
Less 100	−0.15 (−0.89)	0.1937 (0.66)	−0.1574 (−1.09)	0.0138 (0.17)	0.1969 (0.97)	0.1829 (2.55)*	0.0145 (0.32)
Less 1000	0.2638 (2.36)*	-0.0698 (−0.32)	0.081 (0.86)	0.0707 (1.29)	−0.2484 (−1.44)	−0.0236 (−0.52)	0.009 (0.31)
Less 5000	−0.2237 (−4.15)*	−0.0554 (−0.56)	−0.0073 (−0.17)	0.0649 (2.41)*	0.1963 (2.35)*	0.113 (5.49)*	0.0541 (4.09)*
Standard deviation				– –	0.126 (4.67)	– –	0.028 (2.17)
Congestion	−0.5659 (−5.38)*	−0.1293 (−0.58)	0.2079 (2.21)*	0.539 (12.08)*	0.3015 (1.79)	0.1542 (3.55)*	−0.0228 (−0.76)
Standard deviation				0.298 (5.61)	– –	– –	– –
Capacity reduction	−4.1418 (−11.27)*	−1.0789 (−1.72)	−1.5133 (−6.1)*	0.5694 (3.28)*	0.4315 (1)	0.7469 (5.15)*	0.3438 (4.52)*
				0.108 (4.31)	– –	0.229 (3.16)	– –
Block shoulder	−2.0726 (−13.03)*	−0.2834 (−0.92)	−0.6485 (−4.96)*	0.4753 (6.89)*	0.7354 (3.13)*	0.1625 (2.33)*	0.092 (2.2)*
Bike	−2.3147 (−10.5)*	−2.2452 (−4.4)*	−1.1793 (−9.07)*	0.1475 (0.87)	−0.8456 (−1.67)	−0.1898 (−2.38)*	−0.1724 (−5.56)*
Standard deviation				– –	– –	0.024 (2.81)	0.018 (3.01)
Ambulance	−3.5866 (−19.43)*	−1.2568 (−6.23)*	−2.123 (−22.41)*	0.6876 (5.3)*	0.3303 (1.96)	0.3613 (5.78)*	0.1854 (6.81)*
Standard deviation				0.089 (3.54)	– –	0.076 (4.38)	– –
Fire	−3.0038 (−4.07)*	−0.5056 (−0.81)	−2.1229 (−4.61)*	2.3356 (5.96)*	0.9584 (2.34)*	0.7595 (2.97)*	0.5124 (4.87)*
Standard deviation				0.202 (3.68)	– –	– –	0.064 (3.49)
Injure	−3.095 (−7.59)*	−1.5698 (−3.91)*	−1.1654 (−8.14)*	0.3705 (1.33)	0.5219 (1.5)	0.9262 (11.03)*	0.0791 (2.82)*
Standard deviation				– –	– –	0.198 (5.27)	0.013 (2.97)
Taxi	−0.4397 (−1.99)*	−0.9933 (−1.63)	0.1714 (1.04)	0.1289 (1.08)	0.4403 (0.75)	−0.0335 (−0.44)	0.0759 (1.53)
Bus	−0.5691 (−1.61)	−0.5072 (−0.68)	−0.3378 (−1.06)	0.2374 (1.34)	1.3055 (3.01)*	0.175 (1.17)	0.3381 (4.28)*
Standard deviation				– –	0.487 (2.01)	– –	0.236 (3.92)
Truck	−2.5012 (−3.29)*	−0.3989 (−0.52)	−0.4446 (−1.28)	1.4225 (1.62)	1.0005 (2.06)*	0.1065 (0.62)	0.1518 (1.46)
Motor	−1.5346 (−10.02)*	−0.8698 (−3.25)*	−1.109 (−9.18)*	0.2919 (3.03)*	0.204 (0.82)	−0.1338 (−1.87)	−0.1864 (−6.33)*
$\hat{\gamma}$				2.3624 (51.44)	2.2112 (15.22)	2.3435 (48.31)	2.615 (64.64)

[a] Standard deviation and t-stat of normally distributed parameter.

injuries, or motorcycles are less likely to be cleared by method 3 than method 4.

As expected, for all types of clearance methods, the results of this study indicate that incidents with capacity reduction, shoulder blocking, or fire had longer duration. If there are more incidents in the 5000 m distance within a time interval, the current incident duration will be longer. These findings are consistent with previous studies (Kim et al., 2008; Vlahogianni and Karlaftis, 2013; Zhan et al., 2011).

For incidents with clearance method 1, if the incidents occurred under congestion condition or with ambulances, fire, or motorcycles, the duration will last longer. In incidents with clearance method 2, if there are more incidents within 5000 m during the same time or if the incidents involved buses, the duration will be longer.

As expected, for clearance method 3, if there are more incidents in the same road or within 100 m or if the incident occurred under congestion condition, the duration will be longer, which may be due to heavy traffic hampering the towing work. Moreover, if the incident involved injuries or needed an ambulance, the duration will tend to be longer. For clearance methods 3 and 4, if bikes had been involved in an incident, the duration will last shorter, because towing a bike is easier than other vehicles.

Finally, for incidents with clearance method 4, if an incident involved an ambulances, fire, buses, or trucks, the incident duration will be longer. However, if a bike had been involved in the incident, the incident will be shorter.

Based on the observed and predicted results, the root mean squared error (RMSE) and MAPE were calculated to investigate the accuracy of the predictions. RMSE and MAPE are defined as follows:

$$\mathrm{RSME} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(O_i - P_i)^2} \tag{18}$$

$$\mathrm{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{O_i - P_i}{O_i}\right| \tag{19}$$

where O_i is the observed duration for ith traffic incident; P_i is the predicted duration for ith traffic incident; and n is the number of traffic incident records to be predicted. Lower RMSE and MAPE values correspond to more accurate predictions by the model. The classical accelerated failure time model was used to compare the prediction performance of different models. Four distributions, namely, Weibull, log-normal, log-logistic, and generalized gamma (GG) distribution, were tested for the AFT models, for five dataset of incidents with different clearance methods.

Table 5 shows the RMSE and MAPE calculation results of two models for two data groups, with corresponding duration.

Similar to a previous study with the same dataset (Pereira et al., 2013), durations less than 15 min are difficult to predict. Table 6 shows that, for shorter durations 2–15 min, both RMSE and MAPE indicate that the developed models cannot provide reasonable prediction results, similar to the results of previous studies (Khattak et al., 2012; Pereira et al., 2013). For durations longer than 15 min, the developed mixture model can provide reasonable predictions in accordance with the Lewis' criteria (Lewis, 1982), since it falls below 0.5; however, the developed AFT model cannot. Total RMSE and MAPE show that the developed mixture model is significantly better than the classical AFT model.

Table 5
RMSE and MAPE of two models.

	Mixture model		Classical AFT model	
Duration range	RMSE	MAPE	RMSE	MAPE
2–15	13.71	1.857	13.68	2.185
>15	31.46	0.454	44.35	0.504
Total	26.61	0.947	37.23	1.096

Table 6
Summary of evaluation of prediction accuracy.

Performance measure	Mixture model	Classical AFT model
≤15 min	60.43%	59.46%
≤30 min	82.39%	79.12%
≤60 min	95.29%	93.76%
Mean prediction error	17.71 min	20.15 min

Another measure of effectiveness frequently used in traffic incident duration prediction is related to a given tolerance of the prediction error (Chung, 2010; Kim and Chang, 2012; Smith and Smith, 2001; Zhan et al., 2011). In this study, three tolerance values were used: 15, 30, and 60 min. Table 6 shows the results for this measure of the model. The performance measures "≤15 min", "≤30 min", and "≤60 min" indicate that the error range between observed and predicted total time durations are within a 15, 30, and 60 min, respectively. Moreover, 60.83% of the total duration of incidents was predicted with less than 15 min of prediction error. Table 6 shows that approximately 5% of the prediction absolute error is larger than 60 min, which may be the effect of factors not used in this study, such as the road alignment, traffic flow volume, and weather condition.

6. Conclusion

This study presented a mixture model for competing risks data to analyze and predict incident duration based on two years of incident records from Singapore. All records were used to develop the mixture model for analyzing the various factors that may influence the probability and duration of different clearance methods. Two-thirds of the dataset were then used to develop the mixture model for predicting incident duration, and the rest were used to test the prediction accuracy.

Empirical analysis shows that different characteristics of incidents have significant effect on the clearance methods and duration. For example, an incident involving injured people is more likely to be cleared by clearance method 4 (with tow and police simultaneously) and the duration will be longer than those without injured people. The findings highlight that an appropriate model should be adopted in analyzing traffic incident duration because of the existence of different clearance methods.

On the other hand, the model estimation results show that the random parameter model is better than the fixed parameter model, which highlights that it is important to consider the unobserved heterogeneity in the traffic incident duration models. The results also indicate that the durations of incidents with different clearance method are different and result from different factors. A related note concerns potential endogeneity associated with certain clearance methods (e.g., police), as there may be circular causality (e.g., long duration accidents may demand presence of police, which in turn may imply a longer clearance process). The available data does not provide clear information on this issue, but it should be taken into consideration in future developments.

RMSE and MAPE show that the developed competing risks mixture model can provide reasonable predictions for incidents with duration longer than 15 min, which are better than the prediction results of the classical AFT model. These findings can aid in predicting the duration of traffic incidents with different clearance methods accurately and in managing traffic incident clearance processes efficiently. For the extremely short duration range (i.e., 2–15 min), however, the developed model cannot provide reasonable prediction results, which is consistent with previous studies.

A concern with parametric methods used in this study is whether the adopted distribution appropriately describes the dataset characteristics. Three kinds of distributions were tested in this study; however, a flexible parameter using a piecewise hazard function may give greater flexibility, but is dependent only on the number of defined intervals. How to adopt the most appropriate distribution remains an important area for future research.

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