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# Risk Aversion in Travel Mode Choice with Rank-Dependent Utility 

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# RISK AVERSION IN TRAVEL MODE CHOICE WITH RANK-DEPENDENT UTILITY 

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Using 2004 stated preference data on travel mode collected in the Zürich area, different parametric specifications of the rank-dependent utility function in a logit mixture model show that commuters are weakly averse to small-time losses. The results also justify Yaari's dual theory of choice under risk, that the utility function is linear on outcomes but that the perception of corresponding probabilities is biased. For leisure travel, the travelers are risk neutral to small losses of time.

Keywords: mixed multinomial logit model; rank dependent utility theory; risk aversion; risky choices

JEL codes: C25; D12

## 1. INTRODUCTION

Variability of travel time introduces risk for travelers in that they do not know exactly when they arrive. Reliability of travel conditions affect travel choices. However, few travel choice models take travel time variability explicitly into account because of lack of data, competing theories on choice behavior under risk, or lack of testable empirical models.

Criticism addressed to the expected utility theory (EUT; von Neumann and Morgenstern, 1944) for the modeling of decision under risk has led to nonexpected utility theories (Starmer, 2000). Among theories applied to analysis of risky travel choices, EUT, rank-dependent utility theory (RDU, proposed initially under the denomination of anticipated utility theory by Quiggin, 1982), cumulative prospect theory (Tversky and Kahneman, 1992), and search-theoretic and random regret minimization models (Chorus et al., 2008, 2010) are the most common. EUT still prevails. For example, Noland and Small (1995) and Noland et al. (1998) considered

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travel time variability in the choice of departure time, where utility is defined as a scheduling cost (Small, 1982). Palma and Picard $(2005,2006)$ estimated the distribution of risk aversion toward travel time variability in the choice of itinerary. They tested different utility functions, each corresponding to a pattern of risk aversion as in Pratt (1964) and Arrow (1970): decreasing absolute risk aversion, constant relative risk aversion, and so on. However, Avineri and Prashker (2004) pointed out that experimental data often violate the EUT.

The attention given to an outcome should not depend only on the probability of the outcome but also on the comparison with other possibilities (Diecidue and Wakker, 2000). RDU accounts for rank dependence. It accommodates several empirical violations of EUT such as Allais' paradox and distinguishes between weak and strong risk aversion (in the sense of Pratt, 1964; Rothschild and Stiglitz, 1970, 1971; Arrow, 1970). While risk aversion in EUT corresponds to a simple condition on the shape of the utility function, the RDU model disentangles the perception of probabilities from attitudes toward the outcomes. The functional representation of preferences is defined as the combination of two functions, one for the evaluation of the outcomes and the other for the perception of the probability distribution of these outcomes. A major consequence is a more realistic modeling of risk aversion (Quiggin, 1993; Cohen, 1995). In travel choice analysis, Lapparent (2010) used a RDU model to estimate the choice of air route using data on revealed preference.

Cumulative prospect theory enhances RDU, in that gains and losses are evaluated with respect to a reference point. Gao et al. (2010) chose the free flow (no congestion) travel times of routes as reference points (thereby falling back on RDU). In an experiment about the choice of route (one risky against one safe), Katsikopoulos et al. (2000) used the travel time of the safe route as the reference point. Ben-Elia et al. (2008) and Ben-Elia and Shiftan (2010) made the reference point change from time to time due to experience and information. The definition of a reference point is an unresolved issue as there is seldom an a priori belief whose measure is more suitable. Unfortunately, many applications lack a convenient reference point.

We focus on the effects of travel time variability on the choice of travel mode for work and leisure trips in the Zürich metropolitan area. We propose alternative specifications of discrete choice models based on the RDU theory and use them to characterize and measure risk aversion. Using 2004 data on stated preference, each alternative being characterized by its price and a two-outcome travel time prospect, ten specifications of the RDU function are estimated and compared for each trip purpose. The econometric formulation is based on mixtures of logit models (Train, 2003).

## 2. MODEL

### 2.1. Framework

$\mathcal{D}_{n, s}:=\left\{\mathcal{L}_{n, s, 1}, \ldots, \mathcal{L}_{n, s, J}\right\}$ is a discrete set of $J$ alternatives to which decision maker $n$ is faced in choice situation $s$. An alternative $\mathcal{L}_{n, s, j}$ characterizes a travel mode $j$. It consists of a bundle of one risky and several deterministic attributes. The deterministic attributes $\left\{x_{n, s, j}, \xi_{n, s_{j} j}\right\}$ represent observed attributes (including the travel cost), and the deterministic attributes $\xi_{n, s, j}$ represent unobserved attributes, independent of $x_{n, s, j}$ and of the risky attribute, which is travel time. The latter takes
two possible values: $\underline{t}_{n, s, j}$ with probability $1-p_{n, s, j}$ and $\bar{t}_{n, s, j}$ with probability $p_{n, s, j}$, $p_{n, s, j} \in[0,1], 0<\underline{t}_{n, s, j}<\bar{t}_{n, s, j}$. The travel time outcomes are ordered from best to worst. $p_{n, s, j}$ is the probability of experiencing the worst travel time outcome. An alternative is defined as:

$$
\begin{equation*}
\mathcal{L}_{n, s, j}:=\left\{\underline{t}_{n, s, j}, \bar{t}_{n, s, j}, p_{n, s, j}, x_{n, s, j}, \xi_{n, s, j}\right\} . \tag{1}
\end{equation*}
$$

The pairwise comparison operator $\succeq_{z_{n}, \zeta_{n}}$ is defined on $\left(\mathcal{D}_{n, s}, \Omega_{\mathcal{D}_{n, s}}\right)$ (where $\Omega_{\mathcal{D}_{n, s}}$ is a $\sigma$-field on $\mathcal{D}_{n, s}$ ). It models the preferences of decision maker $n$, that is, how $n$ compares and ranks two alternatives drawn from $\mathcal{D}_{n, s}$. This decision maker is characterized by a set of independent observed and unobserved characteristics $\left\{z_{n}, \zeta_{n}\right\} . \zeta_{n}$ and $z_{n}$ are independent.

Quiggin (1982), Yaari (1987), Segal (1989), Wakker (1994), and Chateauneuf (1999) gave necessary and sufficient conditions under which there exists a RDU representation of preferences. We adapt this formulation as:

$$
\begin{equation*}
U_{z_{n}, \zeta_{n}}\left(\mathcal{L}_{n, s, j}\right)=V_{z_{n}, \zeta_{n}}\left(\underline{t}_{n, s, j}, \bar{t}_{n, s, j}, p_{n, s, j}, x_{n, s, j} ; \theta\right)+\epsilon_{n, s, j} \tag{2}
\end{equation*}
$$

where $V_{z_{n}, \zeta_{n}}$, the systematic part of the utility function, depends on the observed attributes and on a vector of parameters $\theta$, which we shall estimate. $\epsilon_{n, s, j}=\epsilon\left(\xi_{n, s, j}\right)$ is a transitory idiosyncratic shock that is uncorrelated with $V_{z_{n}, \zeta_{n}}$, itself defined as:

$$
\begin{align*}
& V_{z_{n}}, \zeta_{n}\left(\underline{t}_{n, s, j}, \bar{t}_{n, s, j}, p_{n, s, j}, x_{n, s, j} ; \theta\right)=z_{n}^{\prime} \beta_{j}+\sigma_{j} \eta_{n}+x_{n, s, j}^{\prime} \gamma \\
& \quad-\left(v\left(\underline{t}_{n, s, j} ; \delta\right)+\psi\left(p_{n, s, j} ; \alpha\right)\left(v\left(\bar{t}_{n, s, j} ; \delta\right)-v\left(\underline{t}_{n, s, j} ; \delta\right)\right)\right) . \tag{3}
\end{align*}
$$

$z_{n}^{\prime} \beta_{j}$ stands for the linear effects of the observed characteristics of the traveler. It is specific to the alternative $j . \eta_{n}=\eta\left(\zeta_{n}\right)$ is a one-way (individual) random persistent unobserved heterogeneity effect from one choice situation to another. $x_{n, s, j}^{\prime} \gamma$ stands for the contribution of the observable deterministic attributes of the $j$-th alternative to the utility.

The expression in parentheses is the RDU travel time disutility function. It includes the disutility of the best time outcome and weighs the possibility of delay by the perceived probability of the worst travel time outcome. $\psi$ and its curvature reflect the perception of probabilities, "optimism" or "pessimism." $v$ and its curvature reflect the marginal disutility of the travel time outcome.

In this specification of the utility, the weights of the observed and unobserved attributes do not depend on the observed and unobserved characteristics of the traveler. Unobserved characteristics of the traveler weigh additively and independently in the utility function as alternative specific one-way effects.

### 2.2. RDU and Risk Aversion

Risk aversion in the RDU framework is accounted simultaneously for the evaluation of outcomes by the function $v$ and for the perception of probabilities by the function $\psi$ (Chew et al., 1987; Yaari, 1987; Chateauneuf and Cohen, 1994; Cohen, 1995, in press). A linear $\psi$ reduces the RDU framework to the EUT frame-
work as long as $v$ is strictly convex. A strictly convex $v$ and a strictly concave $\psi$ characterize strong risk aversion. A linear $v$ and a strictly concave $\psi$ characterize weak risk aversion. It is also possible that a strictly convex $v$ and a strictly convex $\psi$ may coexist and characterize weak risk aversion: the decision maker is sufficiently optimistic toward the probability of the bad travel time outcome to compensate strong dislike of the longer travel time.

### 2.3. Functional Form of $v$

One functional form for $v$ is:

$$
\begin{equation*}
v\left(t_{n, s, j} ; \delta\right)=\delta_{1} \exp \left(-\delta_{2} t_{n, s, j}\right), \quad \delta_{1} \geq 0, \delta_{2} \leq 0 \tag{4}
\end{equation*}
$$

It is a convex disutility function. It means that the traveler strongly dislikes longer travel times. Any increase in travel time is evaluated as more than proportional to the value of the increase. The constraints on the signs of the parameters $\delta_{1}$ and $\delta_{2}$ ensure that $v$ is a disutility function.

A second functional form is a quadratic function:

$$
\begin{equation*}
v\left(t_{n, s, j} ; \delta\right)=\delta_{1} t_{n, s, j}+\delta_{2} t_{n, s, j}^{2}, \quad \delta_{1} \geq 0, \delta_{2} \geq 0 \tag{5}
\end{equation*}
$$

The constraints on the signs of the parameters $\delta_{1}$ and $\delta_{2}$ ensure that $v$ is a disutility function. It is also a convex function expressing the fact that the traveler strongly dislikes longer travel times. $\delta_{2}=0$ turns $v$ into the linear specification $\delta_{1} t_{n, s, j}$. In such a case, the traveler is said to have a neutral attitude toward travel time outcomes.

A third specification of the disutility function allows for three special cases and is defined as a Box-Cox transformation (Box and Cox, 1964) of the travel time:

$$
\begin{equation*}
v\left(t_{n, s, j} ; \delta\right)=\delta_{1} \frac{t_{n, s, j}^{\delta_{2}}-1}{\delta_{2}}, \quad \delta_{1} \geq 0, \delta_{2} \geq 0 \tag{6}
\end{equation*}
$$

An attractive feature of this specification is that $\delta_{2}=1$ turns $v$ into the linear specification $\delta_{1} t_{n, s, j}$, and $\delta_{2}=0$ turns it into the logarithmic specification $\delta_{1} \ln \left(t_{n, s, j}\right) . \delta_{2} \in[0,1[$ makes the travel time disutility function concave and means that the traveler slightly dislikes longer travel times: an increase in travel time generates a less than proportional additional disutility. $\delta_{2}>1$ makes the time disutility function convex.

### 2.4. Functional Form of $\psi$

The probability weighting function $\psi$ is modeled by means of either a power transformation (PT),

$$
\begin{equation*}
\psi\left(p_{n, s, j} ; \alpha\right)=p_{n, s, j}^{\alpha}, \quad \alpha>0 \tag{7}
\end{equation*}
$$

or a Tversky and Kahneman (1992) inverted $S$-shape transformation,

$$
\begin{equation*}
\psi\left(p_{n, s, j} ; \alpha\right)=\frac{p_{n, s, j}^{\alpha}}{\left(p_{n, s, j}^{\alpha}+\left(1-p_{n, s, j}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \quad \alpha>0 \tag{8}
\end{equation*}
$$

The power transformation allows only for either overweighting (pessimism) or underweighting (optimism) of the probability of the bad travel time outcome, whereas the Tversky and Kahneman transformation allows simultaneously for overweighting of "small" values of the probability (those located below a threshold $\bar{p}(\alpha)$ ) and underweighting "large" values of the probability (those located above this threshold). Many other transformation functions exist ( Wu and Gonzalez, 1996; Prelec, 1998). Stott (2006) made an inventory of functional forms.

### 2.5. Distributions of Unobserved Terms

The $\epsilon$ s are independently and identically Gumbel distributed. Their joint cumulative distribution function is defined as:

$$
\begin{equation*}
F(\epsilon)=\prod_{n=1}^{N} \prod_{s=1}^{T} \prod_{j=1}^{J} \exp \left(-\exp \left(-\mu \epsilon_{n, s, j}\right)\right) \tag{9}
\end{equation*}
$$

where $\mu$ is the scale of the distribution.
The distribution of the vector of individual random effects, $\eta_{n}$, is defined as the product of independent standard normal distributions across individuals:

$$
\begin{equation*}
\forall n, \eta_{n} \xrightarrow{\mathrm{iid}} \mathcal{N}(0,1) . \tag{10}
\end{equation*}
$$

Finally, the $\epsilon$ s and the $\eta \mathrm{s}$ are independently distributed.

### 2.6. Probability of Choice

For a given $n$, we observe $\underline{t}_{n}, \bar{t}_{n}, p_{n}, x_{n}, z_{n}$, and a sequence of choices. Let $y_{n, s, j}=1$ if traveler $n$ has chosen the alternative $j$ in situation $s$, and 0 otherwise. $y_{n}$ is the vector of observed choices for traveler $n$.

The distribution of $\eta_{n}$ and $\epsilon_{n}$ determines the joint probability distribution of a sequence of choices conditional on the observed independent variables: $t_{n}, \bar{t}_{n}, p_{n}, x_{n}$, $z_{n}$. In the RDU framework, the probability of a sequence of $T$ choices is defined as the probability that, in each choice situation, the selected alternative maximizes his utility:

$$
\begin{align*}
& \operatorname{Pr}\left(n \text { chooses } j_{1}, \cdots, j_{T} \mid t_{n}, \bar{t}_{n}, p_{n}, x_{n}, z_{n}\right)= \\
& \operatorname{Pr}\left(\forall s, \forall m_{s} \neq j_{s}, U_{z_{n}, \zeta_{n}}\left(\mathcal{L}_{n, s, j}\right)>U_{z_{n}, \zeta_{n}}\left(\mathcal{L}_{n, s, m}\right) \mid \underline{t}_{n}, \bar{t}_{n}, p_{n}, x_{n}, z_{n}\right) . \tag{11}
\end{align*}
$$

For the sake of clarity, we rewrite $V_{z_{n}, \zeta_{n}}\left(t_{n, s, j}, \bar{t}_{n, s, j}, p_{n, s, j}, x_{n, s, j} ; \theta\right) \equiv V_{z_{n}}$ $\left(\underline{n}_{n, s, j}, \bar{t}_{n, j, j}, p_{n, s, j}, x_{n, s, j}, \eta_{n} ; \theta\right)$. Under assumptions of Eq. (9) and (10), it is a mixture of products of logit choice probabilities:
$\operatorname{Pr}\left(y_{n} \mid \underline{t}_{n}, \bar{t}_{n}, p_{n}, x_{n}, z_{n} ; \theta\right)=$
$\int_{-\infty}^{+\infty}\left(\prod_{s=1}^{T} \prod_{j=1}^{J}\left(\frac{\exp \left(\mu V_{z_{n}}\left(\underline{t}_{n, s, j}, \bar{t}_{n, s, j}, p_{n, s, j}, x_{n, s, j}, \eta ; \theta\right)\right)}{\sum_{k=1}^{J} \exp \left(\mu V_{z_{n}}\left(\underline{t}_{n, s, k}, \bar{t}_{n, s, k}, p_{n, s, k}, x_{n, s, k}, \eta ; \theta\right)\right)}\right)^{y_{n, s j}}\right) \varphi(\eta) \mathrm{d} \eta$,
where $y_{n, s, j}$ is equal to 1 if $j$ is chosen by $n$ in situation $s$. It is equal to 0 otherwise. The $\log$-likelihood function for a sample of $N$ individuals is then:

$$
\begin{equation*}
\ell(\theta \mid y, \underline{t}, \bar{t}, p, x, z)=\sum_{n=1}^{N} \ln \left(\operatorname{Pr}\left(y_{n} \mid \underline{t}_{n}, \bar{t}_{n}, p_{n}, x_{n}, z_{n} ; \theta\right)\right) \tag{13}
\end{equation*}
$$

Set $\beta_{j}=0$ for an arbitrary $j, \mu=1$ and $\sigma_{j}=0$ (Ben-Akiva and Lerman, 1985; Walker et al., 2007, for the identification of parameters in discrete choice models) to obtain a one-to-one mapping between the set of parameters to estimate and the value of the log-likelihood function. The estimation is made using simulation-based inference (Train, 2003) as the latter has no closed-form solution.

We estimate the parameters of interest by maximizing the simulated loglikelihood function

$$
\begin{equation*}
\ell_{\operatorname{sim}}(\theta \mid y, \underline{t}, \bar{t}, p, x, z, \hat{\eta})=\sum_{n=1}^{N} \ln \left(\frac{1}{R} \sum_{r=1}^{R} \operatorname{Pr}\left(y_{n} \mid \underline{t}_{n}, \bar{t}_{n}, p_{n}, x_{n}, z_{n}, \hat{\eta}_{n}^{r} ; \theta\right)\right), \tag{14}
\end{equation*}
$$

where

$$
\begin{align*}
& \operatorname{Pr}\left(y_{n} \mid t_{n}, \bar{t}_{n}, p_{n}, x_{n}, z_{n}, \hat{\eta}_{n}^{r} ; \theta\right)= \\
& \prod_{s=1}^{T} \prod_{j=1}^{J}\left(\frac{\exp \left(\mu V_{z_{n}}\left(\underline{t}_{n, s, j}, \bar{t}_{n, s, j}, p_{n, s, j}, x_{n, s, j}, \hat{\eta}_{n}^{r} ; \theta\right)\right)}{\sum_{k=1}^{J} \exp \left(\mu V_{z_{n}}\left(\underline{t}_{n, s, k}, \bar{t}_{n, s, k}, p_{n, s, k}, x_{n, s, k}, \hat{\eta}_{n}^{r} ; \theta\right)\right)}\right)^{y_{n, s j}} \tag{15}
\end{align*}
$$

For each traveler $n$, and given values of $\theta, R$ draws of $\eta_{n}$ are taken from the probability density function $\varphi$. For each draw, the joint probability in Eq. (15) is then calculated and the results are averaged over draws. The objective is then to maximize the simulated log-likelihood function $\ell_{\text {sim }}$ over $\theta$. The same draws are reused every time necessary to compute $\ell_{\text {sim }}$ and its derivatives. If all draws are independent from one another and from the probability in Eq. (15), then the simulated probability converges almost surely to the "true" probability, with variance inversely proportional to $R$. In the estimation of the maximum simulated likelihood, if $R$ increases faster than the square root of the total number of observations, then the effects of simulation disappear asymptotically, and the maximum simulated likelihood (MSL) is equivalent to the maximum likelihood with exact probabilities. Under these regularity conditions (and those related to the standard maximum likelihood estimator), the MSL estimator is asymptotically unbiased, consistent, normal, and efficient.

## 3. DATA

Data are drawn from a 2004 stated preference survey on travel mode choice conducted in the Zürich area. The first population consists of travelers who undertook a work trip over 10 kilometers long. The second population consists of travelers who undertook a leisure trip over 10 kilometers long.

We consider the private and the public modes of transport. For each individual in each sample, the choice experiments were built from a previously reported actual trip during an earlier travel survey. Vrtic et al. (2005) have described the collecting of the data.

Table 1. Descriptive statistics, work purpose
Total number of observations: 907, total number of individuals: 114

|  |  | Standard <br> Leviation | Minimum | Maximum | Frequency |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Decision Maker characteristics |  |  |  |  |  |
| Age | 39.23 | 11.67 | 16 | 63 | - |
| Gender: man | - | - | - | - | 70 |
| Holding travel card | - | - | - | - | 59 |
| Car availability | - | - | - | - | 67 |
| Choices of individuals | - | - | - | - |  |
| Private transport (PRT) | - | - | - | - | 342 |
| Public transport (PUT) |  |  |  |  |  |
| Attributes of stated preference scenarios | 4.79 | 3.50 | 1.30 | 27.00 | - |
| Cost in Swiss Francs, PRT | 0.50 | 0.24 | 0.08 | 1.67 | - |
| Time in hours, best, PRT | 0.19 | 0.08 | 0.10 | 0.30 | - |
| Probability of being 10 minutes late, PRT | 4.47 | 2.87 | 0.80 | 26.50 | - |
| Cost in Swiss Francs, PUT | 1.29 | 0.49 | 0.30 | 3.28 | - |
| Time in hours, best, PUT | 0.12 | 0.12 | 0.00 | 0.30 | - |
| Probability of being 10 minutes late, PUT | 0.87 | 0.88 | 0.00 | 3.00 | - |
| Total number of transfers |  |  |  |  | - |

Each of the 110 commuters in the first sample responded to eight choice experiments. Three individuals responded to only seven experiments, and one individual responded to only six experiments. The sociodemographic characteristics of the sampled travelers include age, gender, travel card ownership, and car availability.

Table 2. Descriptive statistics, leisure purpose
Total number of observations: 1862, total number of individuals: 235

| Label | Mean | Standard <br> deviation | Minimum | Maximum | Frequency |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Choices of individuals |  |  |  |  |  |
| Age | 47.39 | 16.94 | 16 | 80 | - |
| Gender: man | - | - | - | - | 129 |
| Worker | - | - | - | - | 157 |
| Holding travel card | - | - | - | - | 93 |
| Car availability | - | - | - | - | 151 |
| Decision Maker choices | - | - | - | - | 1288 |
| Private transport (PRT) | - | - | - | - | 574 |
| Public transport (PUT) |  |  |  |  |  |
| Attributes of stated preference scenarios | 13.10 | 12.07 | 1.3 | 65.90 | - |
| Cost in Swiss Francs, PRT | 1.05 | 0.96 | 0.08 | 7.83 | - |
| Time in hours, best, PRT | 0.17 | 0.10 | 0.05 | 0.30 | - |
| Probability of being 10 minutes late, PRT | 14.19 | 14.32 | 1 | 79 | - |
| Cost in Swiss Francs, PUT | 2.18 | 1.35 | 0.23 | 7.85 | - |
| Time in hours, best, PUT | 0.10 | 0.10 | 0 | 0.25 | - |
| Probability of being 10 minutes late, PUT | 1.76 | 1.32 | 0 | 6 | - |
| Total number of transfers |  |  |  | - |  |

Of the respondents, $61.4 \%$ are men. Age ranges from 16 to 63 with an average value of 39.2 years. Possession of a travel card and car availability are used to measure the baseline inclination of a traveler to select a public or a private travel mode of transport. Table 1 presents descriptive statistics.

Each of the 235 travelers in the second sample responded to eight choice experiments. One individual responded to only four experiments, two to only six experiments, and ten to only seven experiments. The sociodemographic characteristics of the sampled travelers also include age, gender, employment status, travel card ownership, and car availability. Of the respondents, $54.9 \%$ are men. Age ranges from 16 to 80 with an average value of 47.4 years. $66.8 \%$ of the respondents are workers. Table 2 presents descriptive statistics.

Each travel mode in the choice experiments is characterized by a bundle of risky and deterministic attributes. The latter are the travel cost and the total number of transfers. The risky attribute, travel time, has two outcomes: the "good" outcome is travel time under free-flow (no congestion) conditions, and the "bad" outcome is defined as the free flow travel time plus 10 minutes. The probability that the latter happens is given.

## 4. RESULTS

For identification purpose, the baseline alternative is the private mode of transport, and the baseline decision maker is a woman. Approximations of integrals are made using 1000 Halton draws (Train, 2003).

### 4.1. Work Purpose

The estimates (and their $t$-statistics in parentheses) for the 10 alternative specifications are presented in Tables 3 and 4.

The one-way random effect is statistically significant. It indicates the presence of persistent unobserved heterogeneity from one choice situation to another, thereby creating serial correlation. It means that some unobserved characteristics of the decision maker or omitted variables contribute significantly to the explanation of observed choices.

The results also show that whatever the formulation of the RDU disutility function, a traveler owning a travel card is more likely to select the public mode of transport. Similarly, the probability that a traveler chooses the public mode is significantly lower when a car is available.

Gender plays no role. A quadratic formulation of age was found to play no significant role, but a quadratic formulation of age in terms of difference from a reference age is significant. There is a significant positive effect of this difference on the probability of the public mode.

The probability of mode choice is a decreasing function of its travel cost: the larger the travel cost by a mode, the lower the probability of choosing it. The results show that the total number of transfers has a significant negative effect on the probability of the public mode.

With regard to the RDU disutility functions, the exponential travel time disutility function (forcing the traveler to strongly dislike longer travel times) yields

| Label | Exponential | Quadratic | Box-Cox transformation | Linear | Logarithmic |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.75 (0.80) | 0.79 (0.90) | 0.71 (0.91) | 0.64 (0.86) | 0.79 (1.19) |
| Age - 40 in years | 0.05 (1.68) | 0.05 (1.87) | 0.05 (1.93) | 0.05 (1.79) | 0.05 (2.17) |
| (Age - 40) ${ }^{2}$ in years | -0.0006 (-0.30) | $-0.0006(-0.30)$ | -0.0006 (-0.30) | -0.0006 (-0.30) | -0.001 (-0.53) |
| Gender: man | 0.76 (1.29) | 0.76 (1.28) | 0.77 (1.31) | 0.78 (1.33) | 0.54 (0.92) |
| Owns a travel card | 3.59 (4.87) | 3.60 (4.87) | 3.58 (4.85) | 3.59 (4.90) | 3.31 (4.73) |
| Usually has a car available | -1.33 (-2.24) | -1.33 (-2.24) | -1.33 (-2.25) | -1.35 (-2.29) | -1.26 (-2.12) |
| Cost | -0.45 (-3.51) | -0.45 (-3.50) | -0.45 (-3.52) | -0.46 (-3.58) | -0.42 (-3.49) |
| Total number of transfers | -1.05 (-4.78) | -1.06 (-4.77) | -1.05 (-4.78) | -1.04 (-4.76) | -1.09 (-5.13) |
| Individual random effect: $\sigma_{\text {PT }}$ | 2.63 (6.33) | 2.63 (6.34) | 2.62 (6.30) | 2.59 (6.31) | 2.59 (6.65) |
| Rank-dependent utility theory time disutility function (v) |  |  |  |  |  |
| Time: $\delta_{1}$ | 24.50 (0.72) | -3.88 (-2.59) | -3.20 (-5.15) | -3.15 (-5.34) | -2.66 (-4.61) |
| Time: $\delta_{2}$ | -0.16 (-0.58) | -0.07 (-0.70) | 0.90 (3.52) | - | - |
| Probability weighting function $(\psi)$ <br> $\alpha$ | 0.51 (2.94) | 0.51 (2.95) | 0.51 (2.91) | 0.51 (2.83) | 0.15 (1.43) |
| Goodness-of-fit |  |  |  |  |  |
| Total number of parameters | 12 | 12 | 12 | 11 | 11 |
| Log-likelihood value at convergence | -345.53 | -345.52 | -345.65 | -345.60 | -353.85 |
| Log-likelihood value at 0 | -628.68 | -628.68 | -628.68 | -628.68 | -628.68 |
| Log-likelihood value, intercept only | -611.30 | -611.30 | -611.30 | -611.30 | -611.30 |
| Adjusted pseudo- $\rho^{2}$ | 0.43 | 0.43 | 0.43 | 0.43 | 0.42 |
| Akaike Information Criterion | 703.06 | 703.04 | 703.30 | 702.20 | 718.70 |
| Bayesian Information Criterion | 772.78 | 772.76 | 773.02 | 766.11 | 782.61 |

Table 4. Probability to choose public transport, work purpose, maximum simulated likelihood estimates, II

| Label | Exponential | Quadratic | Box-Cox transformation | Linear | Logarithmic |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.43 (0.60) | 0.82 (1.06) | 0.73 (0.93) | 0.66 (0.95) | 0.75 (0.91) |
| Age - 40 in years | 0.08 (3.09) | 0.05 (1.87) | 0.05 (1.88) | 0.05 (1.92) | 0.05 (1.78) |
| $\left(\right.$ Age - 40) ${ }^{2}$ in years | -0.0002 (-0.13) | $-0.0006(-0.30)$ | -0.0006 (-0.30) | -0.0006 (-0.28) | -0.001 (-0.58) |
| Gender: man | 1.18 (1.76) | 0.76 (1.29) | 0.77 (1.31) | 0.79 (1.34) | 0.51 (0.88) |
| Owns a travel-card | 3.64 (5.87) | 3.58 (4.88) | 3.57 (4.86) | 3.58 (4.89) | 3.24 (4.64) |
| Usually has a car available | -1.24 (-2.25) | -1.33 (-2.24) | -1.33 (-2.26) | -1.33 (-2.27) | -1.24 (-2.08) |
| Cost | -0.47 (-4.15) | -0.45 (-3.53) | -0.46 (-3.56) | -0.46 (-3.61) | -0.42 (-3.52) |
| Total number of transfers | -1.01 (-4.57) | -1.05 (-4.76) | -1.04 (-4.77) | -1.04 (-4.76) | -1.09 (-5.16) |
| Individual random effect: $\sigma_{\text {PT }}$ | 2.97 (6.17) | 2.62 (6.34) | 2.61 (6.30) | 2.60 (6.26) | 2.60 (6.74) |
| Rank-dependent utility theory time disutility function (v) |  |  |  |  |  |
| Time: $\delta_{1}$ | 24.00 (1.13) | -3.79 (-2.57) | -3.16 (-5.14) | -3.11 (-5.29) | -2.15 (-4.40) |
| Time: $\delta_{2}$ | -0.15 (-0.94) | -0.07 (-0.66) | 0.91 (3.69) | - | - |
| Probability weighting function $(\psi)$ : Tversky-Kahneman transformation |  |  |  |  |  |
| $\alpha$ | 3.14 (3.73) | 0.85 (12.47) | 0.85 (12.31) | 0.85 (12.05) | 0.82 (15.00) |
| Goodness-of-fit |  |  |  |  |  |
| Total number of parameters | 12 | 12 | 12 | 11 | 11 |
| Log-likelihood value at convergence | -345.61 | -346.03 | -346.16 | -346.22 | -356.40 |
| Log-likelihood value at 0 | -628.68 | -628.68 | -628.68 | -628.68 | -628.68 |
| Log-likelihood value, intercept only | -611.30 | -611.30 | -611.30 | -611.30 | -611.30 |
| Adjusted pseudo- $\rho^{2}$ | 0.43 | 0.43 | 0.43 | 0.43 | 0.42 |
| Akaike Information Criterion | 703.22 | 704.06 | 704.32 | 703.44 | 723.80 |
| Bayesian Information Criterion | 772.94 | 782.41 | 782.67 | 767.35 | 787.71 |

no convincing results. Even though the signs of the parameters $\delta_{1}$ and $\delta_{2}$ are correct, their $t$-statistics suggest that the RDU disutility function has no significant effect on the choice of mode. This specification is the only one resulting in an optimistic perception of the probability of longer travel time for either the power transformation or for the Tversky and Kahneman (1992) transformation with $\psi$ being strictly convex. Thus, travelers underestimate the probability of the longer travel time outcome.

The unconvincing results of the exponential formulation may be due to its excessively high degree of convexity. That is why we estimate a quadratic formulation of the travel time disutility function. The estimates have all the expected signs, but the parameter $\delta_{2}$, which stands for the curvature of $v$, is not significant. The travelers are neutral in their evaluation of longer travel time but $\psi$ is found to be concave, denoting a pessimistic perception of the probability of the "bad" travel time outcome. When the probability weighting function is the Tversky and Kahneman (1992) transformation, the estimated threshold probability above which they become optimistic is about 0.45 whereas the maximum probability of being 10 minutes late which is proposed in the stated choice experiments is 0.30 .

As strong dislike for longer travel time is called into question, we turn to the Box-Cox specification of the travel time disutility function. In addition to significant and negative effects of the travel time attribute on the probability to choose the public mode, the Box-Cox parameter $\delta_{2}$ is about 0.9 and significantly different from 0 whatever the specification of the $\psi$ function. We check that assuming weak dislike for longer travel time is inappropriate by estimating a logarithmic specification of $v$. The results show that it has the least goodness of fit. Moreover, the Box-Cox parameter $\delta_{2}$ is not significantly different from 1 whatever the specification of $\psi$. This supports the results derived from the estimation of the quadratic formulation of the travel time disutility function and both suggest a linear specification of $v$.

With $v$ linear, the estimate related to the travel time prospect has the expected sign and is significant. $\psi$ is strictly concave whatever the chosen probability weighting function. Using likelihood ratio tests, we do not reject a linear specification of $v$ for all the tested specifications of $\psi$. Using either the Akaike information criterion (AIC) or the Bayesian information criterion (BIC) to select the most adequate model, we conclude that a linear $v$ and a concave power probability weighting function $\psi$ are the most adequate specifications of how travel time prospects are perceived by travelers.

Yaari's (1987) dual theory of choice under risk performs better. Travelers are weakly risk averse. They prefer average travel time to travel time prospect. Also, they evaluate travel time linearly and are pessimistic about the probability of longer travel time.

### 4.2. Leisure Purpose

The estimates (and their $t$-statistics in parentheses) for the ten alternative specifications are presented in Tables 5 and 6.

In all specifications, the one-way random effect is statistically significant, but age, gender, and employment status play no significant role on the probability of mode choice. Travel card ownership increases significantly the probability of public mode whereas car availability decreases it. The results also show that the travel cost
Table 5. Probability to choose public transport, leisure purpose, maximum simulated likelihood estimates, I

| Label | Exponential | Quadratic | Box-Cox transformation | Linear | Logarithmic |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1.94 (3.33) | 1.95 (3.25) | 1.90 (3.32) | 1.83 (3.15) | 1.62 (2.90) |
| Age - 50 in years | 0.02 (1.23) | 0.02 (1.21) | 0.02 (1.23) | 0.02 (1.28) | 0.01 (1.11) |
| $(\text { Age }-50)^{2}$ in years | 0.0001 (0.20) | 0.0001 (0.20) | 0.0002 (0.23) | 0.0002 (0.24) | 0.0003 (0.41) |
| Gender: man | 0.08 (0.25) | 0.08 (0.25) | 0.09 (0.26) | 0.09 (0.29) | 0.06 (0.19) |
| Owns a travel-card | 1.05 (3.25) | 1.05 (3.25) | 1.05 (3.24) | 1.05 (3.22) | 1.16 (3.77) |
| Usually has a car available | -1.72 (-4.84) | -1.72 (-4.84) | -1.72 (-4.84) | -1.72 (-4.83) | -1.62 (-4.82) |
| Is a worker | -0.35 (-0.79) | -0.35 (-0.79) | -0.34 (-0.77) | -0.33 (-0.75) | -0.31 (-0.74) |
| Cost | -0.14 (-6.10) | -0.14 (-6.14) | -0.14 (-6.10) | -0.14 (-6.10) | -0.11 (-5.39) |
| Total number of transfers | -0.58 (-6.71) | -0.58 (-6.71) | -0.58 (-6.65) | -0.57 (-6.57) | -0.65 (-8.05) |
| Individual random effect: $\sigma_{\text {PT }}$ | 2.04 (9.41) | 2.04 (9.42) | 2.05 (9.40) | 2.06 (9.44) | 1.87 (9.52) |
| Rank-dependent utility theory time disutility function (v) |  |  |  |  |  |
| Time: $\delta_{1}$ | 29.1 (0.80) | -1.59 (-4.27) | -1.46 (-5.21) | -1.39 (-8.10) | -1.82 (-5.53) |
| Time: $\delta_{2}$ | -0.06 (-0.68) | -0.02 (-0.79) | 0.94 (6.20) | - | - |
| Probability weighting function $(\psi)$ : power transformation |  |  |  |  |  |
| $\alpha$ | 0.83 (2.17) | 0.83 (2.17) | 0.81 (2.14) | 0.81 (2.18) | -0.13 (-1.50) |
| Goodness-of-fit |  |  |  |  |  |
| Total number of parameters | 13 | 13 | 13 | 12 | 12 |
| Log-likelihood value at convergence | -782.21 | -782.23 | -782.47 | -782.51 | -804.40 |
| Log-likelihood value at 0 | -1290.64 | -1290.64 | -1290.64 | -1290.64 | -1290.64 |
| Log-likelihood value, intercept only | -1150.18 | -1150.18 | -1150.18 | -1150.18 | -1150.18 |
| Adjusted pseudo- $\rho^{2}$ | 0.38 | 0.38 | 0.38 | 0.38 | 0.37 |
| Akaike Information Criterion | 1577.42 | 1577.46 | 1577.94 | 1577.02 | 1620.80 |
| Bayesian Information Criterion | 1662.30 | 1662.34 | 1662.82 | 1655.37 | 1699.15 |

Table 6. Probability to choose public transport, leisure purpose, maximum simulated likelihood estimates, II

| Label | Exponential | Quadratic | Box-Cox transformation | Linear | Logarithmic |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1.90 (3.20) | 1.95 (3.28) | 1.90 (3.20) | 1.85 (3.17) | 1.54 (2.68) |
| Age - 50 in years | 0.02 (1.22) | 0.02 (1.21) | 0.02 (1.22) | 0.02 (1.22) | 0.01 (0.98) |
| $\left(\right.$ Age -50) ${ }^{2}$ in years | 0.0002 (0.21) | 0.0001 (0.19) | 0.0002 (0.23) | 0.0002 (0.24) | 0.0004 (0.58) |
| Gender: man | 0.07 (0.23) | 0.08 (0.25) | 0.08 (0.26) | 0.09 (0.27) | 0.05 (0.17) |
| Owns a travel-card | 1.07 (3.29) | 1.05 (3.25) | 1.05 (3.24) | 1.05 (3.21) | 1.21 (3.96) |
| Usually has a car available | -1.73 (-4.87) | -1.72 (-4.84) | -1.72 (-4.84) | -1.72 (-4.84) | -1.62 (-4.83) |
| Is a worker | -0.35 (-0.78) | -0.35 (-0.79) | -0.34 (-0.78) | -0.35 (-0.77) | -0.32 (-0.76) |
| Cost | -0.14 (-6.60) | -0.14 (-6.14) | -0.14 (-6.10) | -0.14 (-6.12) | -0.11 (-5.34) |
| Total number of transfers | -0.58 (-6.75) | -0.58 (-6.71) | -0.58 (-6.65) | -0.57 (-6.57) | -0.66 (-8.23) |
| Individual random effect: $\sigma_{\text {PT }}$ | 2.04 (9.39) | 2.04 (9.42) | 2.05 (9.39) | 2.06 (9.40) | 1.86 (9.63) |
| Rank-dependent utility theory time disutility function (v) |  |  |  |  |  |
| Time: $\delta_{1}$ | 19.5 (1.58) | -1.58 (-4.28) | -1.46 (-5.27) | -1.38 (-8.11) | -1.34 (-4.76) |
| Time: $\delta_{2}$ | $-0.09(-1.20)$ | -0.02 (-0.79) | 0.94 (6.29) | - | - |
| Probability weighting function $(\psi)$ : Tversky-Kahneman transformation |  |  |  |  |  |
| $\alpha$ | 8.28 (8.53) | 1.07 (4.98) | 1.06 (5.03) | 1.07 (5.12) | 0.65 (9.92) |
| Goodness-of-fit |  |  |  |  |  |
| Total number of parameters | 13 | 13 | 13 | 12 | 12 |
| Log-likelihood value at convergence | -781.90 | -782.24 | -782.49 | -782.60 | -810.49 |
| Log-likelihood value at 0 | -1290.64 | -1290.64 | -1290.64 | -1290.64 | -1290.64 |
| Log-likelihood value, intercept only | -1150.18 | -1150.18 | -1150.18 | -1150.18 | -1150.18 |
| Adjusted pseudo- $\rho^{2}$ | 0.38 | 0.38 | 0.38 | 0.38 | 0.36 |
| Akaike Information Criterion | 1576.80 | 1577.48 | 1577.98 | 1577.20 | 1632.98 |
| Bayesian Information Criterion | 1661.68 | 1662.36 | 1662.86 | 1655.55 | 1711.33 |

and the total number of transfers have significant and negative effects on the probability of choosing a public mode.

With regard to the RDU specification of the disutility function, the best representation of observed choice is a linear specification of $v$ and a linear specification of $\psi$.

The exponential formulation of the time disutility function yields no convincing result. Its parameters are not significant; and the travelers pay no attention to travel time. When we define $v$ as a logarithmic function (forcing the traveler to dislike weakly longer travel times), the results show that the two models (one for each of the two proposed functions $\psi$ ) are the least explanatory. When we use the Box-Cox or the quadratic formulation of the time disutility function, the results and some statistical tests show that both can be reduced to a linear travel time disutility function.

Looking at the probability weighting functions excluding the models with the exponential and the logarithmic specifications of travel time disutility, the estimates are not statistically different from 1. EUT is appropriate to modeling how travel time prospects are evaluated.

In sum, travelers are neutral to risk of small time losses when considering a delay of 10 minutes with some probability. They evaluate travel time prospect accounting only for the corresponding expected travel time.

Using either the AIC or the BIC to select the most adequate model, one should select the linear specification of the time disutility function and the power weighting function.

## 5. CONCLUSION

The estimates show that the von Neumann and Morgenstern (1944) EUT framework is inappropriate to the choice of travel mode for work but appropriate to the choice of travel mode for leisure. A robust result is that Yaari's (1987) dual theory of choice under risk is a sound framework for the analysis of choice of a travel mode for work trips longer than 10 kilometers. For leisure trips, travelers are found to be neutral to risk of small loss of time.

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